DEEQA Macroeconomics 1

Endogenous Rigidity

Robert Ulbricht

Fall 2016

Readings

- Lucas (1972, JET), "Expectations and the Neutrality of Money"
- Woodford (2003), "Imperfect common knowledge and the effects of monetary policy"

Additional references

- Hamilton (1994), Chapter 13 on Kalman filter

1 Lucas 1972

Setup

- There is a continuum of islands $i \in I = [0, 1]$, each inhabited by a unit mass of young agents.
- Agents live for two periods, overlapping in generations. When young, agents work and accumulate money (the only asset in this economy). When old, they travel to another (random) island to spend their money and consume.
- Young agents produce

$$y_{i,t} = n_{i,t}$$

units of log-output per unit of log-labor, yielding a nominal log-income

$$m_{i,t} = p_{i,t} + y_{i,t}$$

- Money is inflated by an exogenous nominal subsidy from the government that is proportional to each agents money holdings $m_{i,t}$:

$$m_{i,t+1} = m_{i,t} + g + \epsilon_{t+1}, \qquad \epsilon_t \sim \mathcal{N}(0, \kappa_\epsilon^{-1})$$

- Integrating over agents, money follows a random walk with drift g

$$m_{t+1} = m_t + g + \epsilon_{t+1}$$

- The sample of old agents that retires in a particular island is representative with random log-mass $\mu_{i,t} \sim \mathcal{N}(0, \kappa_\mu^{-1})$. Nominal demand in island i is thus given by

$$c_{i,t} + p_{i,t} = m_t + \mu_{i,t} = m_{t-1} + g + \mu_{i,t} + \epsilon_t.$$

- Young agents maximize

$$\mathbb{E}\left\{e^{c_{i,t+1}} - \frac{1}{2}e^{2n_{i,t}}|\mathcal{I}_{i,t}\right\},$$

where

$$\begin{aligned} c_{i,t+1} &= m_{i,t+1} - p_{j,t+1} \\ &= p_{i,t} + n_{i,t} + g + \epsilon_t - p_{j,t+1}, \end{aligned}$$

when retiring on island j

- The information of young agents is given by

$$\mathcal{I}_{i,t} = \{m_s\}_0^{t-1} \cup \{p_{i,t}\}.$$

(no need to specify info for old agents as their demand is inelastic to information)

Equilibrium

- $\{p_{i,t}\}$ clears all markets ($c_{i,t} = y_{i,t}$ for all i, t), and young agents choose $n_{i,t}$ optimally given $\mathcal{I}_{i,t}$

- Look for an equilibrium where $p_{i,t}$ is linear. Guess:

$$p_{i,t} = m_{t-1} + g + \beta(\epsilon_t + \mu_{i,t}) \quad \text{for some } \beta \in \mathbb{R}$$

- The FOC of young agents is

$$\begin{aligned} n_{i,t} &= \mathbb{E}\{p_{i,t} - p_{j,t+1} + g + \epsilon_{t+1} | \mathcal{I}_{i,t}\} \\ &= p_{i,t} + g - \mathbb{E}\{p_{j,t+1} | \mathcal{I}_{i,t}\}, \end{aligned}$$

or, after substituting for our conjecture for $p_{i,t}$,

$$\begin{aligned} n_{i,t} &= p_{i,t} + g - \mathbb{E}\{g + m_t + \beta(\epsilon_{t+1} + \mu_{i,t+1})|\mathcal{I}_{i,t}\} \\ &= p_{i,t} - \mathbb{E}\{m_t|\mathcal{I}_{i,t}\}, \end{aligned}$$

where

$$\mathbb{E}\{m_t|\mathcal{I}_{i,t}\} = m_{t-1} + g + \mathbb{E}\{\epsilon_t|\mathcal{I}_{i,t}\}.$$

- Given the conjecture for $p_{i,t}$, prices are a linear Gaussian signal about ϵ_t that is informationally equivalent to observing

$$\hat{p}_{i,t} \equiv \frac{p_{i,t} - g - m_{t-1}}{\beta} = \epsilon_t + \mu_{i,t}.$$

Since m_{t-1} is orthogonal to ϵ_t, we hence have

$$\mathbb{E}\{\epsilon_t|\mathcal{I}_{i,t}\} = \underbrace{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\mu^2}}_{\equiv\chi} \hat{p}_{i,t}.$$

- Imposing market clearing:

$$\underbrace{p_{i,t} - \mathbb{E}\{m_t|\mathcal{I}_{i,t}\}}_{\text{supply}} = \underbrace{m_t + \mu_{i,t} - p_{i,t}}_{\text{demand}},$$

or

$$\begin{aligned} p_{i,t} &= m_t + \frac{1}{2}\left(\mathbb{E}\{m_t|\mathcal{I}_{i,t}\} - m_t + \mu_{i,t}\right) \\ &= m_{t-1} + g + \epsilon_t + \frac{1}{2}\left(\mathbb{E}\{\epsilon_t|\mathcal{I}_{i,t}\} - \epsilon_t + \mu_{i,t}\right) \\ &= m_{t-1} + g + \frac{1}{2}(1+\chi)\left(\epsilon_t + \mu_{i,t}\right), \end{aligned}$$

which confirms our initial guess for $\beta = (1+\chi)/2$

Aggregate implications

- Price level is

$$p_t = m_{t-1} + g + \frac{1}{2}(1+\chi)\epsilon_t$$

- Output is

$$\begin{aligned} y_t &= m_t - p_t \\ &= \frac{1}{2}(1-\chi)\epsilon_t \end{aligned}$$

- Phillips curve

$$\pi_t \equiv p_t - p_{t-1} = g + \frac{1+\chi}{1-\chi}y_t + \frac{1-\chi}{2}\epsilon_{t-1}$$

- In this environment, agents want to produce when they face a high idiosyncratic demand for their output when young ($\mu_{i,t}$), because this increases the real wage for their work. In contrast, when facing a fully known nominal demand shock (ϵ_t), nominal prices adjust equally across all islands, so that the real purchasing power of one unit more work remains unchanged.

- With imperfect information, agents can not fully differentiate between idiosyncratic demand and nominal aggregate shocks, so that in equilibrium agents end up responding to aggregate shocks as well. This creates a short-run non-neutrality of money
  - The larger the "signal-to-noise" ratio χ, the easier it is for agents to distinguish the aggregate from the individual shock, and the less they respond to aggregate shocks.
  - In the limit where $\sigma_\epsilon \to \infty$ or $\sigma_\mu \to 0$, we have that $\chi \to 1$, implying that all changes in nominal demand are absorbed by prices ($\partial p_{i,t}/\partial \epsilon_t = 1$) and output does not respond at all to nominal shocks ($\partial y_{i,t}/\partial \epsilon_t = 0$)
- Lucas critique: effectiveness of monetary policy depends on expectations
  - no response to systematic inflation g, only surprise shocks ϵ have a real impact $\to$ no systematic exploitation of Phillips curve: on average, $\mathbb{E}y_t \propto \mathbb{E}\epsilon_t = 0$
  - Philips curve depends on monetary regime. When σ_ϵ is large, Philips curve becomes vertical (slope in front of y_t becomes ∞)

Summary

- imperfect info affects transmission of monetary shocks
- in particular: sluggish response of prices to m_t yields short-run non-neutrality
- policy regime affects degree of price stickyness and the impact of shocks
- $\bar{\mathbb{E}} m_t \neq m_t$ is key for real effect. Here true for 1 period after money shock
  - What is 1 period? Money supply released within few weeks
  - Stands in contrast to longer responses identified by VARs to money shocks

2 Woodford (2002)

Differences to Lucas

- private noisy signal about nominal demand, but never common knowledge
  - in spirit of Sims (2001)
  - $\bar{\mathbb{E}} m_t \neq m_t$ possibly indefinitely after money shock
- standard monopolistic competition
  - real rigidity adds strategic complementarity in pricing
  - optimal price depends on higher-order expectations, adding additional source of price rigidity

Setup

- simplified setting from last week
  - firms:

$$p_{i,t} = \mathbb{E}[p^*_{i,t}|\mathcal{I}_{i,t}], \qquad p^*_{i,t} = p_t + \xi y_t, \qquad \xi \in (0,1)$$

    where

$$y_t = m_t - p_t$$

  - exogenous money rule:

$$m_t = \rho m_{t-1} + u_t, \qquad \epsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_\epsilon), \qquad \rho \in (-1, 1]$$

- information available to firm i

$$\mathcal{I}_{i,t} = \{s_{i,\tau}\}_{\tau=0}^{t}$$

  where

$$s_{i,t} = m_t + \upsilon_{i,t}, \qquad \upsilon_{i,t} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_\upsilon)$$

Equilibrium

- optimal price is combination of expected average price $p_t = \int p_{i,t}\,\mathrm{d}i$ and expected nominal demand

$$p_{i,t} = \xi \mathbb{E}[m_t|\mathcal{I}_{i,t}] + (1-\xi)\mathbb{E}[p_t|\mathcal{I}_{i,t}]$$

- equilibrium is fixed-point between perceived and actual LOM for p_t

  1. conjecture a perceived LOM for endogenous p_t
  2. compute expectations using Kalman filter
  3. compute optimal decisions and aggregate across agents to get actual LOM of p_t

- guess

$$p_t = \phi_m m_{t-1} + \phi_p p_{t-1} + \phi_\epsilon \epsilon_t$$

- then we can set up the system in state space form:

$$\begin{pmatrix} m_t \\ p_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ \phi_m & \phi_p \end{bmatrix} \begin{pmatrix} m_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{bmatrix} 1 \\ \phi_\epsilon \end{bmatrix} \epsilon_t$$

with the observation equation given by

$$s_{i,t} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} m_t \\ p_t \end{pmatrix} + v_{i,t}$$

- using the Kalman filter, we get

$$\mathbb{E}_{i,t} \begin{pmatrix} m_t \\ p_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ \phi_m & \phi_p \end{bmatrix} \mathbb{E}_{i,t-1} \begin{pmatrix} m_{t-1} \\ p_{t-1} \end{pmatrix} + K_t(s_{i,t} - \mathbb{E}_{i,t-1} m_t)$$

where the 2×1 matrix K_t is the Kalman-gain

- integrating across agents, average expectations are given by

$$\bar{\mathbb{E}}_t \begin{pmatrix} m_t \\ p_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ \phi_m & \phi_p \end{bmatrix} \bar{\mathbb{E}}_{t-1} \begin{pmatrix} m_{t-1} \\ p_{t-1} \end{pmatrix} + K_t(m_t - \bar{\mathbb{E}}_{t-1} m_t)$$

- the actual LOM is given by

$$\begin{aligned} p_t &= \begin{bmatrix} \xi & 1-\xi \end{bmatrix} \bar{\mathbb{E}}_t \begin{pmatrix} m_t \\ p_t \end{pmatrix} \\ &= \xi \rho \bar{\mathbb{E}}_{t-1} m_{t-1} + (1-\xi)\left(\phi_m \bar{\mathbb{E}}_{t-1} m_{t-1} + \phi_p \bar{\mathbb{E}}_{t-1} p_{t-1}\right) + \begin{bmatrix} \xi & 1-\xi \end{bmatrix} K_t(m_t - \bar{\mathbb{E}}_{t-1} m_t) \end{aligned}$$

- note how the price dynamics depend on 2 state variables not present in our initial guess. Woodford's trick is to use the lagged equilibrium condition to substitute out

$$\bar{\mathbb{E}}_{t-1} p_{t-1} = \frac{p_{t-1} - \xi \bar{\mathbb{E}}_{t-1} m_{t-1}}{1-\xi}$$

on the right-hand side. Now we have p_t in terms of p_{t-1}, m_{t-1} and ϵ_t and one additional variable $\bar{\mathbb{E}}_{t-1}m_{t-1}$

$$p_t = \xi\rho\bar{\mathbb{E}}_{t-1}m_{t-1} + (1-\xi)\left(\phi_m\bar{\mathbb{E}}_{t-1}m_{t-1} + \phi_p\frac{p_{t-1} - \xi\bar{\mathbb{E}}_{t-1}m_{t-1}}{1-\xi}\right) + \begin{bmatrix}\xi & 1-\xi\end{bmatrix}K_t(\rho m_{t-1} + \epsilon_t - \rho\bar{\mathbb{E}}_{t-1}m_{t-1})$$

- matching coefficients, we need

$$\phi_m = \begin{bmatrix}\xi & 1-\xi\end{bmatrix}K_t\rho \tag{1}$$

$$\phi_\epsilon = \begin{bmatrix}\xi & 1-\xi\end{bmatrix}K_t \tag{2}$$

$$\phi_p = \phi_p$$

and further that

$$\xi\rho + (1-\xi)\left(\phi_m - \phi_p\frac{\xi}{1-\xi}\right) = \begin{bmatrix}\xi & 1-\xi\end{bmatrix}K_t\rho \tag{3}$$

where the last equation ensures that $\bar{\mathbb{E}}_{t-1}m_{t-1}$ drops out

- we have 4 equations in 3 unknowns. Luckily, the third condition is always true so that we can use the additional degree of freedom to solve the remainder equations. Solving (1)–(3) gives us a mapping from K_t to $(\phi_m, \phi_p, \phi_\epsilon)$.

- To complete the analysis we also need to derive an explicit expression for K_t. To make it time-invariant, suppose that learning has converged to it's steady state so that $K_t = K$, where K is a function of $(\phi_m, \phi_p, \phi_\epsilon)$. The equilibrium is thus given by the fixed point between the two mappings. See Woodford for an analytical solution in the case where $\rho = 1$. Alternatively, use standard root-solver to find the fixed-point

  1. pick ϕ_m, ϕ_p and ϕ_ϵ
  2. compute K_t and solve the Ricatti equation to get the steady state version K
  3. given K, solve (1)–(3) for $(\tilde{\phi}_m, \tilde{\phi}_p, \tilde{\phi}_\epsilon)$
  4. repeat until $(\phi_m, \phi_p, \phi_\epsilon) \approx (\tilde{\phi}_m, \tilde{\phi}_p, \tilde{\phi}_\epsilon)$

Results

- for some $\hat{k} \equiv [\xi, 1-\xi]K$ have that

$$\phi_m = \rho\hat{k}$$

$$\phi_\epsilon = \bar{k}$$

$$\phi_p = \rho(1-\hat{k})$$

so that

$$p_t = \hat{k}\rho m_{t-1} + (1-\hat{k})\rho p_{t-1} + \hat{k}\epsilon_t$$

$$= \hat{k}m_t + (1-\hat{k})\rho p_{t-1}$$

and

$$\begin{aligned} y_t &= m_t - p_t \\ &= (1 - \hat{k})\,(m_t - \rho p_{t-1}) \\ &= (1 - \hat{k})\,(\rho y_{t-1} + \epsilon_t) \end{aligned}$$

- persistence and amplification if y_t depends on $\hat{k}$. Can show that $\hat{k}$ is increasing in ξ, bounded above by 1, and $\hat{k} \to 0$ for $\xi \to 0$. I.e., high degrees of real rigidity (ξ low), the economy's response to nominal shocks can become arbitrarily persistent, independent of the signal-to-noise ratio of firms

Intuition

- rewrite optimality condition

$$
\begin{aligned}
p_{i,t} &= \xi \mathbb{E}_{i,t}\{m_t\} + (1-\xi)\mathbb{E}_{i,t}\{p_t\} \\
&= \xi \mathbb{E}_{i,t}\{m_t\} + (1-\xi)\mathbb{E}_{i,t}\left\{\bar{\mathbb{E}}_t\{\xi m_t + (1-\xi)p_t\}\right\} \\
&\vdots \\
&= \xi \sum_{k=0}^{\infty}(1-\xi)^k \mathbb{E}_{i,t}\left\{\bar{\mathbb{E}}_t^{(k)}\{m_t\}\right\}
\end{aligned}
$$

where

$$
\bar{\mathbb{E}}_t^{(k)} m \equiv \bar{\mathbb{E}}_t \bar{\mathbb{E}}_t^{(k-1)} m_t \qquad \text{and} \qquad \bar{\mathbb{E}}_t^{(0)} m_t \equiv m_t
$$

- optimal response given by hierarchy of higher-order expectations
  - even if a firm knows realization of m_t, it's optimal price response depends on (i) the average expectations regarding m_t and (ii) the average expectation of others regarding the average expectation of others, and so on...
  - strategic complementarity is key

- with strategic complementarity, this is additional source of price sluggishness
  - consider a simple static version of the model with $s_i = m + \varepsilon_i$ where m and $\{\epsilon_i\}$ are independent Gaussian white noise terms
  - then for some $\lambda \in (0, 1)$,

$$\mathbb{E}_i m = \lambda s_i \quad \Longrightarrow \quad \bar{\mathbb{E}} m = \lambda m \quad \Longrightarrow \quad \mathbb{E}_i \bar{\mathbb{E}} = \lambda^2 s_i$$

  - in general

$$\bar{\mathbb{E}}^{(k)} m = \lambda^k m \to 0 \text{ as } k \to \infty$$

  - public info better at predicting what agents mutually agree on, so higher order beliefs become increasingly skewed towards prior (here $\mathbb{E} m = 0$)

3 Limits of state-space approach

- so far, all signals exogenous

- when endogenous signals are present, the state-space approach outlined above runs into a problem: when we include average expectations up to order k in the state space in the conjectured LOM, the actual LOM will depend on average expectations up to order $k+1$. This is Townsend's (1983) problem of forecasting the forecast of others (leading to "infinite regress")

- suppose we conjecture same LOM as above,

$$\begin{pmatrix} m_t \\ p_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ \phi_m & \phi_p \end{bmatrix} \begin{pmatrix} m_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{bmatrix} 1 \\ \phi_\epsilon \end{bmatrix} \epsilon_t,$$

but now have the observation equation

$$s_{i,t} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} m_t \\ p_t \end{pmatrix} + \begin{pmatrix} \upsilon_{i,t} \\ \eta_{i,t} \end{pmatrix}$$

Using the Kalman filter and integrating we get

$$\bar{\mathbb{E}}_t \begin{pmatrix} m_t \\ p_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ \phi_m & \phi_p \end{bmatrix} \bar{\mathbb{E}}_{t-1} \begin{pmatrix} m_{t-1} \\ p_{t-1} \end{pmatrix} + K_t \begin{pmatrix} m_t - \bar{\mathbb{E}}_{t-1} m_t \\ p_t - \bar{\mathbb{E}}_{t-1} p_t \end{pmatrix}.$$

Substituting into the firm's FOCs (and defining K^i as the ith row of K)

$$\begin{aligned} p_t &= \begin{bmatrix} \xi & 1-\xi \end{bmatrix} \bar{\mathbb{E}}_t \begin{pmatrix} m_t \\ p_t \end{pmatrix} \\ &= \xi \rho \bar{\mathbb{E}}_{t-1} m_{t-1} + (1-\xi)\left(\phi_m \bar{\mathbb{E}}_{t-1} m_{t-1} + \phi_p \bar{\mathbb{E}}_{t-1} p_{t-1}\right) + \begin{bmatrix} \xi & 1-\xi \end{bmatrix} K_t^1 (m_t - \bar{\mathbb{E}}_{t-1} m_t) \\ &\quad + \begin{bmatrix} \xi & 1-\xi \end{bmatrix} K_t^2 (p_t - \bar{\mathbb{E}}_{t-1} p_t) \end{aligned}$$

- in red are the new terms due to the endogenous signal $p_t + \eta_{i,t}$.
- Adding the endogenous signal makes the LOM directly dependent on p, so that Woodford's trick no longer works, since the coefficient ϕ_p is now pinned down by the direct dependence on p_t. Hence, we can no longer eliminate $\bar{\mathbb{E}}_{t-1} m_{t-1}$ from the LOM by using ϕ_p

- the dependence on $\bar{\mathbb{E}}_{t-1} m_{t-1}$ is the forecasting the forecast of others problem. If we would add $\bar{\mathbb{E}}_{t-1} m_{t-1}$ to the state-space (so that we would not need to eliminate it from the actual LOM), we then end up instead with $\bar{\mathbb{E}}^{(2)}_{t-1} m_{t-1}$, etc

workarounds to the infinite regress problem

- from before, we know that higher order approximations converge to expectations conditionally on public information alone. This suggests that if we truncate the state space at some high order, the conjectured LOM might approximately match the corresponding LOM (see Nimark 2011 for details)

- Alternatively, note that we could also recursively substitute using the Kalman filter to get

$$\begin{aligned}\bar{\mathbb{E}}_t \begin{pmatrix} m_t \\ p_t \end{pmatrix} &= A\bar{\mathbb{E}}_{t-1} \begin{pmatrix} m_{t-1} \\ p_{t-1} \end{pmatrix} + B \begin{pmatrix} m_t \\ p_t \end{pmatrix} \\ &\vdots \\ &= A^T \bar{\mathbb{E}}_{t-T} \begin{pmatrix} m_{t-T} \\ p_{t-T} \end{pmatrix} + \sum_{s=0}^{T-1} A^s B \begin{pmatrix} m_{t-s} \\ p_{t-s} \end{pmatrix}\end{aligned}$$

  – effectively we “undo” the Kalman filter and instead write agents expectations as a big 1-step updating problem starting with the prior at period $t-T$ and simultaneously updating using all the signals between $t-T$ and t

  – if the prior many periods ago has little importance for (m_t, p_t) today, this suggests that we could also find an approximate solution by writing p_t as a function of lags of m_t and p_t and truncating after some large number of lags. Lorenzoni (2009) adopts this approach.

  – Note how this requires that p_t (and m_t) is stationary (shocks long ago have little impact on the the far-away future, so that also the prior has little importance). Rondina-Walker

2012 challenge this view by showing that with dispersed information one can sometimes construct equilibria in which past shocks never fade out the system

- The previous approach can be made exact without the need of an approximation by instead assuming that all shocks in the economy are revealed with some lag T. Then the first term in the above equation becomes commonly known, so that we can replace $\bar{\mathbb{E}}_{t-T}(m_{t-T}, p_{t-T})$ by (m_{t-T}, p_{t-T}) and write the LOM exactly as a function of the first T lags of m and p. See Hellwig (2002, "Public Announcements, Adjustment Delays and the Business Cycle") for a strategy along this lines

- Finally, in some cases we can solve the fixed point exactly in the frequency domain

  - application of Wiener filter requires fundamental representation
  - can be derived via Blaschke-factorization if number of signals equals number of shocks
  - (also available as side product of Kalman filter, but only really feasible for exogenous signal structures, where typically there is no infinite regress)

4 Using survey data to test for incomplete info